PHYSICS AND ASTROPHYSICS OF BLACK HOLES

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Symbols used in this paper:
v_{esc} is escape velocity; R is radius; M is mass ($M_{\odot}$ is solar mass); c is the speed of light; G is gravitational constant; g is gram; cm is centimetre; ; m is metre; km is kilometre; s is seconds; e is electron; lp is Plank's length

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Escape Velocity of Black Holes

Consider an isolated gravitating body (like the star or planet), let us calculate the escape velocity of the small test object or particle.

Escape velocity

$$V_{esc} = \frac{2GM}{R}$$

Radius of Black Hole (BH)

$$R = \frac{2GM}{V_{esc}}$$

Always $v_{esc} < c$

In limiting case, $v_{esc} = c$, we have a BH

$$R = \frac{2GM}{c}$$

$$G = 6.67\text{x}10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2 = 6.67\text{x}10^{-8}\text{cm}^3/\text{g}\cdot\text{s}^2$$

$$c = 300{,}000\text{km/s} = 3\text{x}10^5\text{km/s} = 3\text{x}10^{10}\text{cm/s}$$

Suppose $M = M_{\odot} = 1.99\text{x}10^{30}\text{kg} = 1.99\text{x}10^{33}\text{g}$
Then solar mass BH has the size

$$R = \frac{(2)6.67\times10^{-8}\,cm^3/g\cdot s^2(1.99\times10^{33}\,g)}{9\times10^{20}\,cm^2/s^2}$$
$$= \frac{2(6.67)1.99}{9}10^{-8+33-10}\,cm$$
$$= \frac{2(6.67)1.99}{9}10^{5}\,cm$$
$$R_{BH} = 2.94\times10^5\,cm \approx 3km$$

Therefore, if our star became a BH, its radius would be ≈ 3km.

In terms of the radius to mass ratio, the more massive the BH, the larger its radius.

Creating Theoretical Black Holes

$$M = \frac{c^2}{2G} R$$

$$R = \frac{2GM}{c^2}$$

Minimal scale in nature
$\geq$ lp = $1.6\text{x}10^{-33}$cm → this is the Plank scale
minimum $M_{BH} \sim 10^{-5}$g

$M_e = 9.1 \text{ x } 10^{-24}$g

$M_p = 1.67 \text{ x } 10^{-24}$g

If an electron became a BH

$$R_{BH} = \frac{2GM_e}{c^2}$$

$$= \frac{(2)6.67 \times 10^{-8} cm^3 / g \cdot s^2 (9.1 \times 10^{-24} g)}{9 \times 10^{20} cm^2 / s^2}$$

$$= \frac{2(6.67)9.1}{9} 10^{-8-28-20} cm$$

$$= \frac{2(6.67)9.1}{9} 10^{-56} cm$$

$$R_{BH} = 13.49 \times 10^{-56} cm$$

$$M_{BH} = \frac{c^2}{2G} R_{BH}$$

$$== \left(\frac{9 \times 10^{20} cm^2 / s^2}{2(6.67 \times 10^{-8} cm^3 / g \cdot s^2)} \right) 13.49 \times 10^{-56} cm$$

$$= \left(\frac{9}{13.34} \right) 13.49 \times 10^{-56+8+20} cm$$

$M_{BH} = 9.1\text{x}10^{-44}$g

Smallest BH

$$M = \frac{c^2}{2G} R$$

$$= \left(\frac{9 \times 10^{20} cm^2 / s^2}{2(6.67 \times 10^{-8} cm^3 / g \cdot s^2)} \right) 1.6 \times 10^{-33} cm$$

$$= \left(\frac{9}{13.34} \right) 1.6 \times 10^{-33+8+20} cm$$

$M = 1.08\text{x}10^{-5}$g

Therefore a BH with a radius of Plank's length will have a mass of about $1.08\text{x}10^{-5}$g

Stellar Black Holes

In order for a star to become a black hole when it dies, is must have a mass equal to or above 25 solar masses. If the star is between 1.4 and 3-5 solar masses, it will become a neutron star. If the star is one solar mass or less, it will become a white dwarf when it dies. There is a battle at death of a heavy star. The pressures of the fleeting nuclear reactions are fighting against the gravity of the dying star. If the gravity overwhelms the pressure, then the core of the star collapses into a black hole. If the pressure equals the gravity, the core collapses to a neutron star. A black hole is 'an object whose surface gravity is so great that no radiation or matter can escape from it. Some black holes discussed in astronomy are collapsed stars, but much smaller ones are theoretically possible.' (The Cosmic Voyage: Through Time and Space; 1992 edition by William K. Hartmann). Black holes don't actually 'suck' matter in; rather their immense gravity causes matter to *fall into* the black hole. The gravitational field causes space-time to warp. It is also so strong that light cannot escape. We cannot "see" the black hole, just its evidence of existence via the accretion disk, matter spiralling into the black hole, and jets of matter. The reason we can see the accretion disk is that the gravity has heated the matter up so much that it glows in different radiation types from radio to gamma wave.

Intermediate Black Holes

Intermediate black holes are about 100-1,000 times more massive than our sun. They originate in open and globular star clusters. They are the result of stars getting too close to each other, merging, and then accumulating enough mass that the object collapses into a black hole. One can think of this process as when one takes a ball of snow, the stars, and compact it so the middle becomes ice, the black hole. The discovery of this class of black hole is fairly recent.

http://www.spaceref.com/tools/vi.html?id=139&cat=blackholes&imgs=movie. This is a movie of a black hole feeding off a blue giant.

http://www.nature.com/news/2004/041108/full/041108-2.html. This link has some information on another black hole in the center of our galaxy. I will get more into supermassive, or galactic, black holes later on.

Supermassive Black Holes

Supermassive black holes have a mass of about 10^6-10^9 $M_\odot$.

Some black holes apparently have nonstellar origins. Various astronomers have speculated that large volumes of interstellar gas collect and collapse into supermassive black holes at the centres of quasars and galaxies. A mass of gas falling rapidly into a black hole is estimated to give off more than 100 times as much energy as is released by the identical amount of mass through nuclear fusion. Accordingly, the collapse of millions or billions of solar masses of interstellar gas under gravitational force into a large black hole would account for the enormous energy output of quasars and certain galactic systems. In 1994 the Hubble Space Telescope provided conclusive evidence for the existence of a supermassive black hole at the centre of the M87 galaxy. It has a mass equal to two to three billion Suns but is no larger than the solar system. The black hole's existence can be strongly inferred from its energetic effects on an envelope of gas swirling around it at extremely high velocities. Similar evidence suggests that a massive black hole with a mass of about 2.6 million Suns lies at the centre of our own Milky Way Galaxy. (http://www.space.com/reference/brit/blackholes/structure.html)